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MODEL OF COLMATAGE-SUFFOSION FILTRATION OF DISPERSE SYSTEMS IN A POROUS MEDIUM

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A model of colmatage-suffosion filtration of disperse systems in a porous medium is analyzed numerically based on a new kinetic equation for saturation of the pore space with settled particles.

Modeling of the phenomena of precipitation of particles from a disperse medium in its filtration in a porous medium and clogging of the pores with these particles (colmatage of the pores) and unclogging of the pores under the action of different forces (suffosion) is of great importance for various technological processes. Some mathematical models of colmatage-suffosion filtration of disperse systems in porous media are given in [1, 2]. These models' drawbacks were discussed in [3, 4]. In the latter works, based on a balance equation for the concentration of particles suspended in the filtered liquid, new kinetic equations of active porosity [5, 6], and a generalized Darcy law a model of colmatage-suffosion filtration that is devoid of the mentioned drawbacks of the models of [1, 2] is proposed and studied. At the same time it is noteworthy that in [2] a balance equation is written in a more general form than in [3, 4], where the change in the saturation of the pore space with the settled particles and the "free" liquid not connected with the settled mass, the change in the volume concentration of the suspended solid substance in the moving mixture, etc. are taken into account separately. In [7] instead of a kinetic equation for the change in the active porosity [5, 6] it is proposed that a kinetic equation for the saturation of the pore space with settled particles in the loose body where allowance is made not only for colmatage and suffosion effects but also for sedimentation precipitation of the particles be used. In [8] a model of colmatage-suffosion filtration of multicomponent systems is proposed and issues of correct formulation of the problem and the existence of invariant and self-similar solutions are discussed. In [9], based on the model of [3, 4], the joint influence of the effects of convective diffusion, colmatage, and suffosion on the concentration of particles in the flow, the active-porosity profiles, the filtration rate, and other filtration characteristics was investigated, and the range of qualitative agreement between the obtained theoretical results and the experimental data given in [2] was determined. Just as in [3, 4], a material-balance equation for particles suspended in a liquid is used as the balance equation in [8, 9]. In this work a model based on the balance equation of [2], the kinetic equation of [7], and a generalized Darcy law is investigated. In light of the foregoing the proposed model takes into account in a more general form the filling of the pore volume with the filtered liquid, the settled particles, and the "free" liquid not connected with the settled mass and takes into consideration at the same time the effects of colmatage, suffosion, sedimentation precipitation of the particles, etc. After the formulation of the model a specific problem is considered whose numerical solution is used as the basis for investigating profiles of saturation of the pore space with settled particles, profiles of the volume concentration of the suspended solid substance in the moving mixture, etc.

We consider the motion of a suspension in a homogeneous porous medium. Here the porous medium and the suspension are such that in the process of the latter's filtration part of the suspension's solid substance is captured by the porous medium, some of the previously settled particles separate and get into the filtration flow, and some are carried by the filtration flow beyond the considered portion.

The filling of unit volume of the pore space in the process of filtration is considered in accordance with the scheme of [2]. Let the saturation of the pore space with the liquid be equal to ρ; then its saturation

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with the solid substance will be $1 - \rho$. The solid particles are in the pore space in either a settled or a suspended state. We denote the saturation of the pore space with the settled mass in the solid body by β and that with suspended particles by α. The saturation of the pore space with settled particles in the loose body equals ζ, and that with the "free" liquid not connected with the settled mass is ξ; we will use δ to denote the volume concentration of the suspended solid substance in the moving mixture. It follows from these definitions that

$$\rho + \alpha + \beta = 1 \quad \text{and} \quad \xi + \zeta + \alpha = 1 ,$$

$$\beta = \zeta - \varepsilon \zeta \quad \text{or} \quad \zeta = \frac{\beta}{1 - \varepsilon} , \tag{1}$$

$$\rho = 1 - \alpha - (1 - \varepsilon) \zeta , \quad \rho = \xi + \varepsilon \zeta .$$

From relations (1) it is clear that to determine the volume concentration and all the saturations, it is sufficient to find δ and ζ as functions of coordinates and time. To do this, we compose a system made up of a balance equation, a kinetic equation, and Darcy's law. We take the balance equation in the same form as in the well-known model of [2], namely,

$$\frac{\partial \delta}{\partial x} = - B(t) \frac{\partial \zeta}{\partial t} - B_1 \frac{\partial \alpha}{\partial t} , \quad B(t) = \frac{m_0 (1 - \varepsilon)}{q(t)} , \quad B_1(t) = \frac{m_0}{q(t)} . \tag{2}$$

Since the intensity of the process of colmatage or suffosion should depend on the averaged values of δ and ζ for the given cross section and the given instant, we will represent the kinetic equation for particle precipitation as

$$\frac{\partial \zeta}{\partial t} = F(\delta, \zeta) . \tag{3}$$

To determine the function $F(\delta, \zeta)$, we will assume that a suspension particle alternately precipitates and separates, i.e., upon getting into the filtration flow the particle will travel over a certain section of the path $l(x)$, then stop or precipitate, move anew, stop again, etc.

Just as in [5, 6], we will describe the kinetics of the particle-precipitation process by a probability method. We consider every pore to represent a trap for mixture particles. Then the porous medium can be considered as a continuum of traps, each of which can be free or occupied. If the trap is occupied, the liquid does not pass through the corresponding pore. We introduce the probabilities of capture w_0 of at least one mixture particle by a trap and its release w_1 per unit time. The probability of capture by pores is determined by both kinematic and dynamic factors. Physical considerations make it clear that colmatage and suffosion processes depend on the pressure gradient, and the greater the value of $|\nabla p|$, the lower the probability of colmatage and the higher the probability of suffosion. On this basis we assume

$$w_0 = \frac{\omega_0}{\eta_0 + \eta_1 |\nabla p|} , \quad w_1 = \omega_1 |\nabla p| , \tag{4}$$

where η_0 characterizes the sedimentation precipitation of the particles in the pores; η_1 characterizes the intensity of the influence of $|\nabla p|$ on colmatage.

The first of equalities (4) reflects the fact that capture of an impurity by pore-traps occurs in the absence of filtration flows, and the second reflects the assumption that unclogging of pores occurs as a result of separation of clogging impurity particles only under the action of the filtration flow.

Using (4), we represent Eq. (3) in the form

$$\frac{\partial \zeta}{\partial t} = \frac{\omega_0}{\eta_0 + \eta_1 |\nabla p|} \delta - \zeta |\nabla p| \omega_1 . \tag{5}$$

The first term on the right-hand side of Eq. (5) characterizes the process of particle precipitation. The intensity of the process of colmatage increases as the concentration δ increases and decreases as $|\nabla p|$ increases. When the flow is absent ($|\nabla p| = 0$) sedimentation precipitation of the particles occurs, which explains the introduction of the parameter η_0. Using the additional parameter η_1, it is possible to vary the intensity of the influence of the pressure gradient ∇p on the process of colmatage. The second term on the right-hand side of (5) reflects the process of separation and washing of the particles out of the pores. It is clear that suffosion is directly dependent on the saturation of the pore space with settled particles ζ. Furthermore, an increase in ∇p causes a rise in the intensity of the particle-separation process.

We rewrite Eq. (5) in the form

$$\frac{\partial \zeta}{\partial t} = \frac{\omega_0}{1 + \gamma |\nabla p|} \delta - \omega_1 |\nabla p| \zeta, \tag{6}$$

where $\gamma = \eta_1/\eta_0$, and ω_0 should hereinafter mean the ratio ω_0/η_0.

With allowance for the limiting suffosion pressure gradient $|\nabla p_s|$ introduced in [10] we can write (6) as

$$\frac{\partial \zeta}{\partial t} = \frac{\omega_0}{1 + \gamma |\nabla p|} \delta - \omega_1 (|\nabla p| - |\nabla p_s|) \zeta. \tag{7}$$

We represent the filtration law in the form

$$v = K(m) (|\nabla p| - |\nabla p_0|), \quad m = m_0 (1 - \beta), \tag{8}$$

where $K(m)$ is the filtration coefficient taken in linear form ($K(m) = k_0 m$, k_0 = const) [5] or in nonlinear form [6].

In (8), unlike [2], the active porosity is considered to decrease only due to the solid particles settled in the pores.

The system of equations (2), (6) (or (7)), and (8) makes up a mathematical model of the process based on which the dynamics of δ, ζ, and other filtration characteristics can be investigated. The remaining parameters will be determined in accordance with (1). Specific problems for this system can be formulated similarly to [2, 8-10].

Considering $q(t)$ in (2) as the filtration rate, we represent this system of equations as

$$\begin{gathered} \frac{\partial \delta}{\partial x} = -B \frac{\partial \zeta}{\partial t} - B_1 \frac{\partial \alpha}{\partial t}, \quad B = \frac{m_0 (1 - \varepsilon)}{v}, \quad B_1 = \frac{m_0}{v}, \\ \frac{\partial \zeta}{\partial t} = \frac{\omega_0}{1 + \gamma |\nabla p|} \delta - \omega_1 |\nabla p| \zeta, \\ v = K(m) (|\nabla p| - |\nabla p_0|). \end{gathered} \tag{9}$$

Since in accordance with (1) $\alpha = \delta(1 - \zeta)$, we can write the first equation of system (9) in a modified form. Then

$$\begin{gathered} \frac{\partial \delta}{\partial x} = -B \frac{\partial \zeta}{\partial t} - B_1 \frac{\partial \delta}{\partial t} (1 - \zeta) + B_1 \delta \frac{\partial \zeta}{\partial t}, \\ \frac{\partial \zeta}{\partial t} = \frac{\omega_0 \delta}{1 + \gamma |\nabla p|} - \omega_1 |\nabla p| \zeta, \quad v = K(m) (|\nabla p| - |\nabla p_0|). \end{gathered} \tag{10}$$

To examine a solution of this system of equations, we consider the following problem. Let a semiinfinite homogeneous bed with the initial porosity m_0 be filled with a homogeneous liquid (i.e., a liquid without solid particles). At the point $x = 0$, starting with $t > 0$, a liquid with a concentration of solid particles δ_0 enters the bed with the filtration rate $v(t) = v_0 = \text{const}$. Then the initial and boundary conditions for the problem have the form

$$\zeta(0, x) = 0 , \quad \delta(0, x) = 0 , \quad \delta(t, 0) = \delta_0 . \tag{11}$$

Thus, the problem is reduced to solving the system of equations (10) with conditions (11).

To solve this problem, we use the finite-difference method. We introduce a grid in the region $D = \{0 \le x < \infty, 0 \le t \le T\}$, where T is the maximum time during which the process is investigated. For this purpose, we subdivide the interval $[0, \infty)$ with a step h and divide $[0, T]$ into J parts with a step τ. As a result we have the grid $\omega_{h\tau} = \{(x_i, t_j), i = 0, 1, ..., j = 0, 1, ..., J, x_i = ih, t_j = j\tau, \tau = T/J\}$. Instead of the functions $\delta(t, x)$, $\zeta(t, x)$, and $|\nabla p(t, x)|$ we will consider grid functions whose values at the nodes (x_i, t_j) will be denoted, respectively, by δ_i^j, ζ_i^j, and $|\nabla p|_i^j$. To approximate the first equation of the system, we select a pattern in the form of a right upper corner [11]. The second and third equations of system (10) will be approximated by explicit finite-difference schemes. As a result we obtain the following system of difference equations for the linear filtration coefficient $K(m) = k_0 m$, $k_0 = \text{const}$:

$$\delta_i^{j+1} = \frac{h\tau}{\tau + B_1 h (1 - \zeta_i^j)} \left[\frac{\delta_{i-1}^{j+1}}{h} + \frac{1}{\tau} (B_1 \delta_i^j - B)(\zeta_i^{j+1} - \zeta_i^j) + \frac{B_1}{\tau} \delta_i^j (1 - \zeta_i^j) \right],$$

$$\zeta_i^{j+1} = \zeta_i^j + \tau \left(\frac{\omega_0 \delta_i^j}{1 + \gamma |\nabla p|_i^j} - \omega_1 |\nabla p|_i^j \zeta_i^j \right), \tag{12}$$

$$|\nabla p|_i^{j+1} = \frac{v_0}{k_0 m_0 (1 - (1 - \varepsilon) \zeta_i^{j+1})} + |\nabla p_0| .$$

The initial and boundary conditions (11) for the grid functions are written in the form

$$\zeta_i^0 = 0 , \quad \delta_i^0 = 0 , \quad \delta_0^j = \delta_0 , \quad i = 0, 1, \dots , \quad j = 0, 1, \dots , J . \tag{13}$$

The computational scheme of the solution is as follows. With account for conditions (13) we determine the values of the grid functions δ_i^j, ζ_i^j, and $|\nabla p|_i^j$ at all points of the zero layer. Next, in accordance with the second equation of (12) we find the values of ξ_i^{j+1} in terms of the known quantities ξ_i^j, δ_i^j, and $|\nabla p|_i^j$ for the corresponding i; by substituting ξ_i^{j+1} into the first equation of (12) δ_i^{j+1} are found in increasing order of change in i; according to the third equation of (12), $|\nabla p|_i^{j+1}$ are determined. The double-layer two-point difference scheme that is used here approximates the initial problem with the first order of accuracy $O(\tau + h)$. The scheme is absolutely stable, which eliminates any limitation on the magnitude of the step τ [11].

As the initial data for the calculations we took $\delta_0 = 0.01$, $m_0 = 0.2$, $\varepsilon = 0.01$, $|\nabla p_0| = 0.01$ MPa/m, and $k_0 = 0.0005$ m^2/(MPa·sec). The values of the other parameters were varied in certain ranges. Using these data we calculated the solution δ, ζ, and $|\nabla p|$ and, based on them, α, β, ρ, and ξ in accordance with relations (1). Some characteristic plots are given in Figs. 1-3.

An analysis of the change in the quantities shows that the values of δ and ζ at the same point of the bed increase with time. In the direction of the depth of the bed they decrease sharply (Fig. 1). With an increase of 10 and 100 times in the parameter γ the calculations show a relative decrease in ζ and an increase in δ at corresponding points of the bed, which confirms theoretical conclusions about the influence of γ on colmatage-suffosion effects [7]. With increase in γ the colmatage of the pores becomes less intense, which decreases the saturation of the pore space with settled particles ζ and increases the concentration of suspended particles in

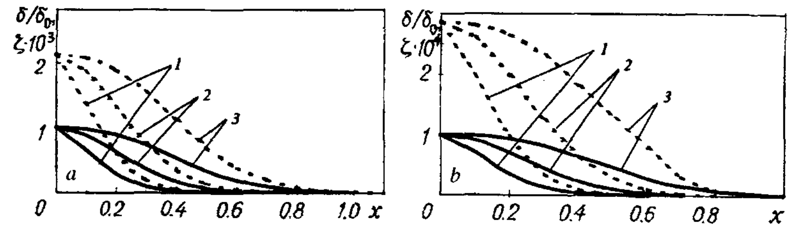

Fig. 1. Profiles of the change in δ/δ_0 (solid curves) and ζ (dashed curves) for $\omega_0 = 0.1$ 1/sec, $\omega_1 = 0.5$ m/(MPa·sec), $\gamma = 10.0$ m/MPa (a), 100.0 (b), $v_0 = 0.25 \cdot 10^{-4}$ m/sec, $t = 10^3$ sec (1), $2 \cdot 10^3$ (2), $3.6 \cdot 10^3$ (3). x, m.

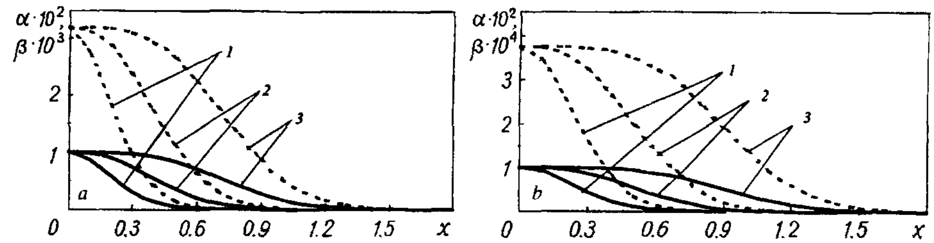

Fig. 2. Profiles of the change in α (solid curves) and β (dashed curves) for $\omega_0 = 0.1$ 1/sec, $\omega_1 = 0.1$ m/(MPa·sec), $\gamma = 10.0$ m/MPa (a), 100.0 (b), $v_0 = 0.5 \cdot 10^{-4}$ m/sec, $t = 10^3$ sec (1), $2 \cdot 10^3$ (2), $3.6 \cdot 10^3$ (3).

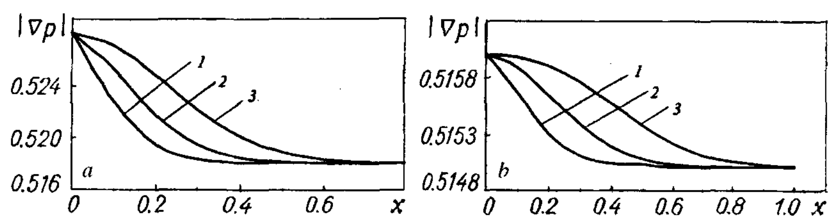

Fig. 3. Profiles of the change in $|\nabla p|$ for $\omega_0 = 0.1$ 1/sec, $\omega_1 = 0.1$ m/(MPa·sec), $\gamma = 0.1$ m/MPa (a), 1.0 (b), $v_0 = 0.5 \cdot 10^{-4}$ m/sec, $t = 10^3$ sec (1), $2 \cdot 10^3$ (2), $3.6 \cdot 10^3$. $|\nabla p|$, MPa/m.

the filtration flow δ. Their profiles become more smeared and mildly sloping. Having increased the filtration rate v_0, we can obtain the profiles of δ and ζ that are advanced noticeably into the bed. Evaluating comparatively the profiles of δ and ζ for different filtration rates v_0, we can notice that with increase in v_0 the concentration of particles in the liquid flow at a fixed point of the bed increases while the saturation of the pore space with settled particles ζ decreases, which indicates a direct influence of v_0 on the colmatage-suffosion effect. Clearly, varying the parameters ω_0 and ω_1, we can also change the characteristics of the filtration process in favor of suffosion or colmatage. Thus, having decreased ω_0 for the same values of v_0, ω_1, γ, and the other parameters, we can notice a decrease in ζ and an increase in δ at a fixed point of the bed. The profiles of α and β (Fig. 2) are similar to the corresponding profiles of δ and ζ (Fig. 1).

Plots of the change in $|\nabla p|$ are shown in Fig. 3. The values of $|\nabla p|$ at fixed points of the bed increase with the running time, which is attributable to the increase in the filtration resistance of the bed due to colmatage effects. With increase in γ the values of $|\nabla p|$ at corresponding points of the bed decrease, which indicates a reduction in the filtration resistance of the bed. The calculations performed showed that, for constant values of the remaining parameters, with increase in v_0 $|\nabla p|$ increases at fixed points of the bed, while an increase in ω_1, for constant values of the other parameters, leads to a decrease in $|\nabla p|$.

By varying the parameters ω_0, ω_1, γ, and others we can investigate different situations where colmatage effects prevail or are negligibly small. Theoretical results obtained based on the model of [3, 4] are compared in [9] with experimental data [2].

In the model that is presented here we do not consider diffusion transfer of particles in the pore space. It can play a significant role [9] in the process of colmatage-suffosion filtration, especially in intense regimes of it. It seems of great interest to compare the influence of diffusion, colmatage, and suffosion effects. Model (9) (or (10)) can be generalized with allowance for effects of particle diffusion in the pore space similarly to [9, 12].

NOTATIONS.

h, step of the grid along the axis $0x$; m_0 and m, initial and current porosity; $q(t)$, total flow rate of the liquid and solid phases per unit area of the porous medium; v_0 and v, prescribed constant and current filtration rate; x, coordinate; δ_0 and δ, constant and current volume concentration of the suspended solid substance in the moving mixture; ε, porosity of the settled mass; γ, parameter that characterizes the intensity of the influence of $|\nabla p|$ on the colmatage; τ, time step of the grid; ω_0 and ω_1, constants that characterize the intensity of the colmatage and the suffosion, respectively; ∇p_0 and ∇p, limiting and current pressure gradients; ∇p_s, limiting suffosion pressure gradient.

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